UNIT- I (Important Questions).

① If $u = (1-2xy+y^2)^{-1/2}$ then Prove that

$$\frac{\partial}{\partial x}\left\{(1-x^2)\frac{\partial u}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{y^2\frac{\partial u}{\partial y}\right\} = 0$$

② If $u = \log(x^3+y^3+z^3-3xyz)$, show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$$

③ If $u = (x^2+y^2+z^2)^{-1/2}$ show that

i) $x u_x + y u_y + z u_z = -u$

ii) $u_{xx} + u_{yy} + u_{zz} = 0$

④ verify that v satisfies $V_{xx} + V_{yy} + V_{zz} = 0$

if ⓐ $V = x^2+y^2-2z^2$

(b) $V = e^{3x+4y}\cos 5x$

⑤ If $u = f(r)$ where $r^2 = x^2+y^2$ then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

⑥ If $u = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2+y^2}$

then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + 2u(x,y) = 0$

⑦ If $u = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$ evaluate $\frac{\partial^2 u}{\partial x \partial y}$

Ans. $\frac{x^2-y^2}{x^2+y^2}$

⑧ If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ then show that

$$x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$$

⑨ If $u = r^n(3\cos^2\theta - 1)$ satisfies the differential eq^n $\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u}{\partial \theta}\right) = 0$ then prove that $n=2$ or -3

Q.10 If $u = x+y+z$, $uv = y+z$, $uvw = z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

Ans $\frac{1}{u^2 v}$

Q.11 If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_1 x_3}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$ then find $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}$

Ans. 4

Q.12 If $x+y+z = u^3+v^3+w^3$, $x^3+y^3+z^3 = u^2+v^2+w^2$, $x^2+y^2+z^2 = u+v+w$ then show that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$$

Q.13 Are the ~~possible~~ function $u = \frac{x-y}{x+z}$, $v = \frac{x+z}{y+z}$ functionally dependent? If so find the relation between them.

Q.14 If $x+y+z = u$, $y+z = u^2 v$, $z = u^2 w$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

Ans u^{-5}

Q.15 If $u = x+y+z$, $v = x^2+y^2+z^2$, $w = x^3+y^3+z^3-3xyz$ Prove that u, v, w are not independent and hence find the relation between them. Ans $w = \frac{u(u^2+3v)}{4}$

Q.16 The temperature T at any point (x,y,z) in space is $T = 400xyz^2$. Find the highest temperature at the surface of a unit sphere $x^2+y^2+z^2 = 1$

Q.17 If $u = ax^2+by^2+cz^2$, where $x^2+y^2+z^2 = 1$ and $lx+my+nz = 0$ prove that stationary values of u satisfy the equation $\frac{l^2}{a-u} + \frac{m^2}{b-u} + \frac{n^2}{c-u} = 0$

Q.18 Find the shortest and longest distance from the point $(1, 2, -1)$ to the sphere. Ans $\sqrt{6}$, $3\sqrt{6}$

Q19 Divide a number into three parts such that the product of first, square of second and cube of third is maximum.

Ans 4, 8, 12

20 A rectangular box which is open at the top has a capacity of 256 cubic feet. Determine the dimension of the box such that the least material is required for the construction of the box.

Ans 8, 8, 4

21 Find the volume of the largest parallelopiped with edges parallel to the axes that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Ans $\frac{8abc}{3\sqrt{3}}$